Deuteron Stripping Reaction on Mg^{26}†

J. SILVERSTEIN, L. J. LIDOFSKY, G. E. MITCHELL, AND R. B. WEINBERG
Columbia University, New York, New York
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The reaction $Mg^{26}(d,p)Mg^{27}$ was studied at several bombarding energies between 3.0 and 5.1 MeV. Angular distributions were measured for the transitions to the ground and first three excited states in Mg^{27}. Absolute spectroscopic factors were calculated using the distorted-wave Born approximation, and the dependence of these results on a lower cutoff radius was investigated. The results are compared to those for the $K=\frac{1}{2}$ band in Mg^{25}.

I. INTRODUCTION

THE properties of the odd-A nuclei in the region near $A=25$ have been described in terms of the single-particle Nilsson model.[1] In particular, the energy spectra, the magnetic moments, $E2$ and $M1$ transition probabilities, and reduced widths have been interpreted in terms of this model with a fair degree of success. A rather intensive theoretical analysis of Mg^{25} has quantitatively described many properties of levels below 2 MeV in excitation, considering them to be members of $K=\frac{5}{2}$ and $K=\frac{1}{2}$ rotational bands.[2] If the deformation of Mg^{27} is similar to that of Mg^{25}, then the rotational model predicts that the spacings and reduced widths of the low-lying levels in Mg^{27} should qualitatively resemble those of the first $K=\frac{1}{2}$ band in Mg^{25}.

These reduced widths for Mg^{27} are the subject of the present investigation. We have studied the reaction $Mg^{26}(d,p)Mg^{27}$ to determine the reduced widths for the formation of the levels of interest. The techniques that were used have been discussed in detail in previous reports.[3] Angular distributions were measured at several deuteron bombarding energies in an attempt to reduce the ambiguities introduced by the presence of any compound nuclear processes. The availability of computer codes employing the distorted-wave Born approximation (DWBA) made it possible to determine absolute spectroscopic factors and to compare these directly with the predictions of the rotational model.

The experimental procedure and apparatus are briefly described in Sec. II, and the experimental results are presented in Sec. III. The details of the DWBA calculations and the optical potentials are discussed in Sec. IV, with particular emphasis on the choice of a cut-off radius for the calculations. Spectroscopic factors are compared with the results of previous work on the reaction and with the prediction of the rotational model in Sec. V. The final section summarizes the results.

II. EXPERIMENTAL TECHNIQUES

A. Apparatus

Magnesium oxide enriched to 97–98% in Mg^{26} was obtained from Oak Ridge National Laboratory. Targets were prepared by the reduction and subsequent evaporation of the magnesium onto thin backings of Formvar or gold. The thickness of the targets obtained by this procedure was approximately 100–200 $\mu g/cm^2$.

The experiment was performed in the 48-in. remote control scattering chamber,[4] using the deuteron beam from the Columbia University 5.5-MeV Van de Graaff accelerator. Reaction protons were observed with a 1-mm-thick surface barrier solid-state detector in conjunction with a 512-channel analyzer and standard detection electronics. In order to remove the elastically scattered deuterons and reaction alpha particles, an array of 0.25-mil aluminum foils was stacked in front of the detector. The use of randomly oriented thin foils minimized any loss of resolution due to local nonuniformities in the aluminum. The over-all resolution of the system was about 100 kVolts.

B. Procedure

Angular distributions for the reaction $Mg^{26}(d,p)Mg^{27}$ were studied at eight bombarding energies in the range 3.0–5.1 MeV. Protons were observed corresponding to transitions to the ground and first three excited states in Mg^{27}. In addition, excitation curves for the four proton groups were measured at 21 and 140°.

By removing the aluminum absorber and observing the deuteron elastic scattering and the reaction protons simultaneously, the ratio of the cross sections for the two processes was determined. Absolute cross sections were obtained by assuming that at a bombarding energy of 3.04 MeV and $\theta_d=20°$, the deuteron elastic scattering from Mg^{26} was purely Rutherford.[3]

C. Carbon Contamination

A typical pulse-height spectrum for the (d,p) reaction is shown in Fig. 1. At $\theta_p=110°$ all of the proton groups of interest are clearly visible, as are the protons arising

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[1] K. H. Bhatt, Nucl. Phys. **39**, 375 (1962).

[2] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960); A. E. Litherland, H. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, Can. J. Phys. **36**, 378 (1958); R. K. Sheline and R. A. Harlan, Nucl. Phys. **29**, 177 (1962).

[3] R. B. Weinberg, G. E. Mitchell, and L. J. Lidofsky, Phys. Rev. **133**, B884 (1964).

[4] R. B. Weinberg, H. M. Dowds, and L. J. Lidofsky, Bull. Am. Phys. Soc. **6**, 307 (1961).

from the reaction $C^{12}(d,p)C^{13}$. The kinematics of the reactions are such that as θ_p was decreased, the carbon group obscured first p_3 and then p_2. This effect is illustrated in Fig. 2. The presence of this carbon contamination in the target made it necessary to measure independently the $C^{12}(d,p)C^{13}$ reaction at the same bombarding energies. Subsequently, its contribution was subtracted from the Mg target spectrum.

D. Data Processing

The output from the multichannel analyzer was punched paper tape. After transferring the data to magnetic tape, the data processing was performed on an IBM-7094 computer. The actual analysis was done with a least-squares-fit code developed at Los Alamos,[5] in which the pulse-height spectrum with background removed is matched by a sum of Gaussians. The details of the data reduction procedure have been presented in a previous report.[3] In Fig. 1, a fit to the pulse-height spectrum is shown.

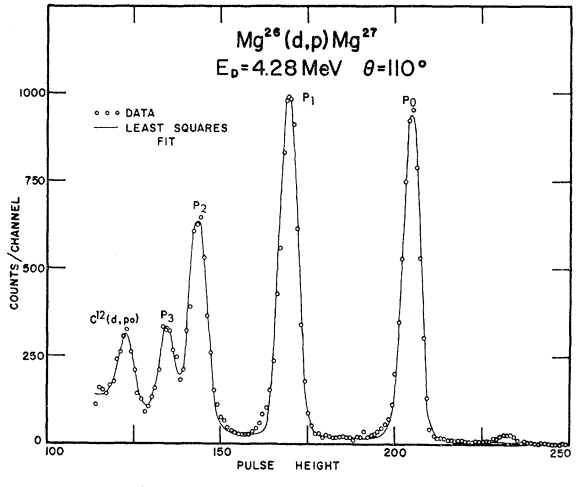

FIG. 1. Pulse-height spectrum for the $Mg^{26}(d,p)Mg^{27}$ reaction. The solid line is the result of a least-squares fit to the data using the Los Alamos programs (Ref. 5).

III. RESULTS

The energy levels for Mg^{27} that are pertinent to this investigation are shown in Fig. 3. Angular distributions were measured for the transition to the ground and first three excited states and are shown in Figs. 4–7. The relative error for the cross sections in the p_0 and p_1 angular distributions is about 5% and is primarily statistical. The p_2 and p_3 angular distributions have an additional uncertainty introduced by the carbon subtraction at forward angles. These forward-angle measurements have a relative error of 15%. The absolute cross sections have an estimated error of 20–25%. This increased error is due in part to the normalization to the deuteron elastic-scattering cross section, and in part to the assumption that the latter have the Rutherford value.

[5] P. McWilliams, W. S. Hall, and H. E. Wegner, Rev. Sci. Instr. **33**, 70 (1962).

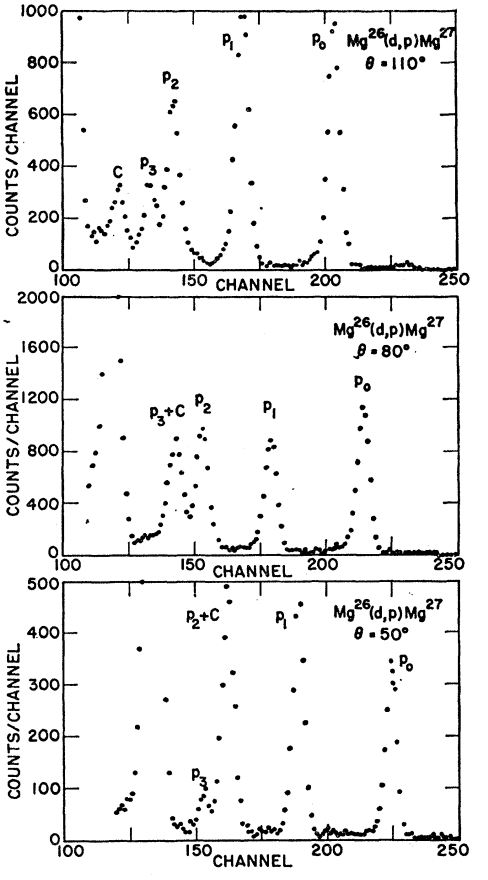

FIG. 2. These spectra illustrate how the proton group from the $C^{12}(d,p_0)C^{13}$ reaction obscures two of the $Mg^{26}(d,p)Mg^{27}$ groups of interest.

With the exception of the third-excited-state transition, the angular distributions behave fairly smoothly as a function of bombarding energy. The strong fluctuations observed in the excitation functions for the p_3 group (Fig. 8) suggest that the variations in the angular distributions are caused by a complex reaction mechanism.

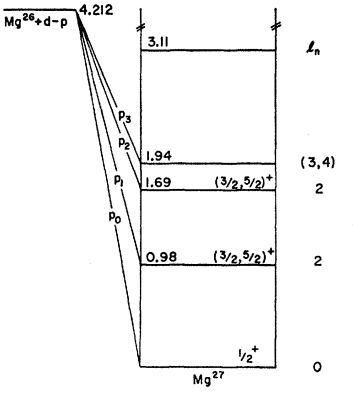

FIG. 3. Low-lying levels of Mg^{27}.

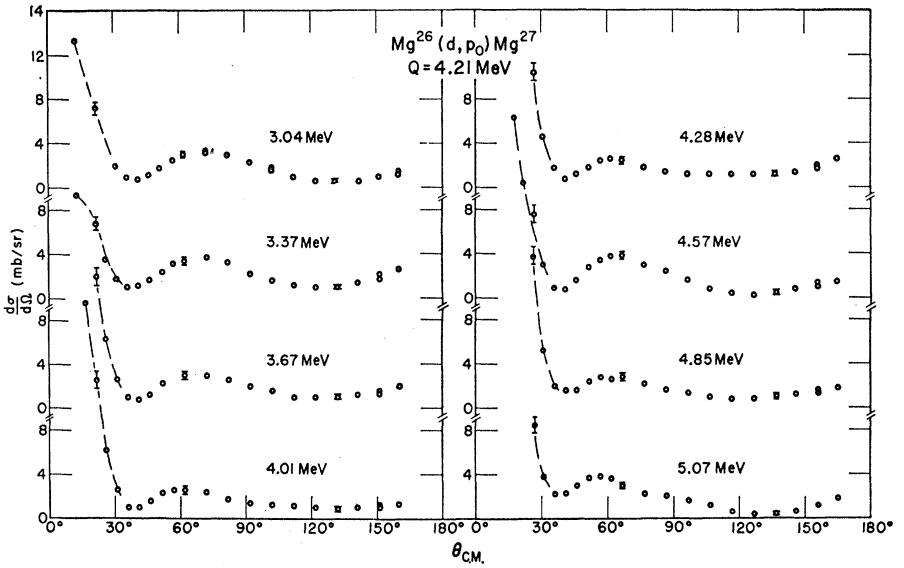

FIG. 4. Angular distributions for the $Mg^{26}(d,p_0)Mg^{27}$ reaction.

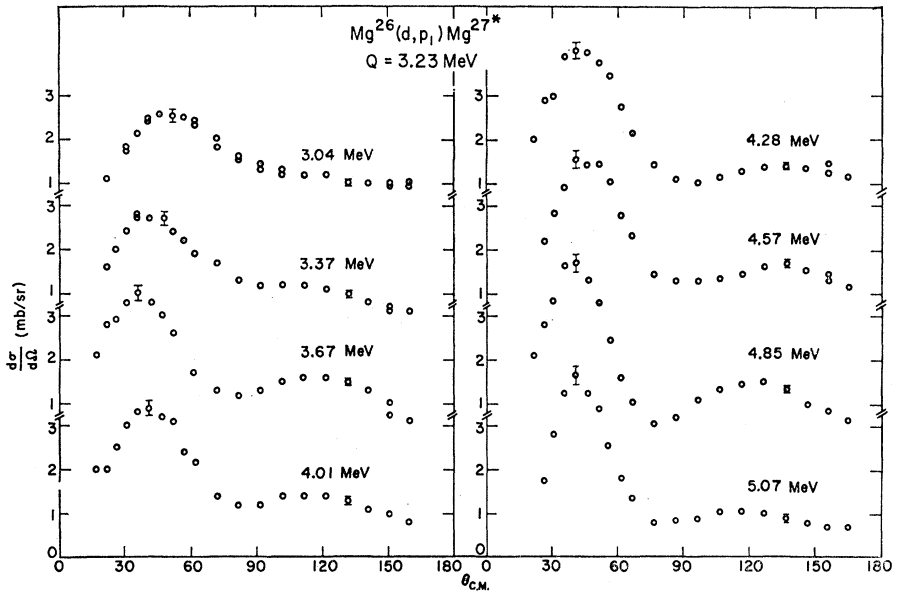

FIG. 5. Angular distributions for the $Mg^{26}(d,p_1)Mg^{27*}$ reaction.

IV. ANALYSIS

A. Optical-Model Parameters

Analysis of the reaction using the distorted-wave Born approximation requires the choice of appropriate optical-model parameters for the entrance and exit channels. The optical potential used in the calculations has the form

$$U=V_{\text{Coulomb}}+V+iW+V_{\text{s.o.}}+iW_{\text{s.o.}},$$

where

$$V=-V_0/(1+e^x), \qquad x=(r-r_0A^{1/3})/a,$$

$$W=-\left(W_0-W_s\frac{d}{dx'}\right)\frac{1}{1+e^{x'}}, \qquad x'=\frac{r-r_sA^{1/3}}{a_s},$$

$$V_{\text{s.o.}}=-V_{\text{s.o.}}\left(\frac{h}{m_\pi c^2}\right)\left[\frac{1}{r}\frac{d}{dr}\left(\frac{1}{1+e^x}\right)\right]\mathbf{l}\cdot\mathbf{s}.$$

The parameters used for the deuteron channel were taken from an investigation performed in this laboratory on 3–5-MeV deuteron elastic scattering from Mg^{25}.[3] It was noted at that time that these values were not unique, and in fact several sets of parameters were found that gave an equivalent fit to the elastic scattering. For the present study the set of parameters given in Table I was used. A quantitative estimate of the effects of variations in these parameters on the reaction calculation is in progress, and will be presented in a future report.

The proton optical potentials are taken from a study by Perey.[6] These potentials were extracted from 9-MeV proton elastic scattering data on Al^{27}. Both sets of

[6] F. G. Perey, Phys. Rev. **131**, 745 (1963).

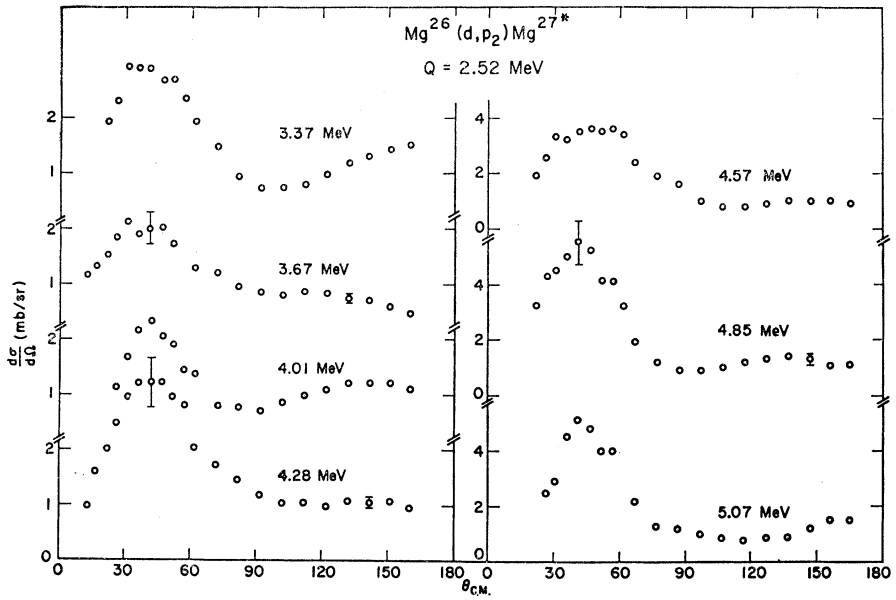

FIG. 6. Angular distributions for the $Mg^{26}(d,p_2)Mg^{27*}$ reaction.

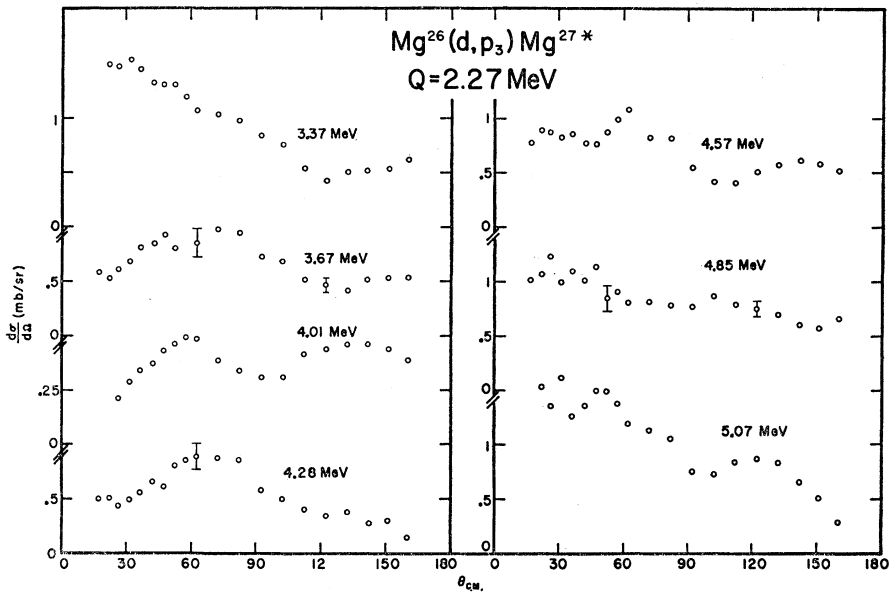

FIG. 7. Angular distributions for the $Mg^{26}(d,p_3)Mg^{27*}$ reaction.

potentials are listed in Table I. Pure surface absorption and no spin-orbit coupling for the deuteron are assumed.

TABLE I. Optical-model parameters used in the DWBA calculations.

	Deuteron	Proton
V (MeV)	75.0	50.9
W_0 (MeV)	0	0
r_0 (F)	1.15	1.25
a (F)	0.81	0.65
W_s (MeV)	60.0	25.4
r_s (F)	1.34	1.25
a_s (F)	0.68	0.47
$V_{s.o.}$ (MeV)	0	7.5

B. Distorted-Wave Born Approximation

The spectroscopic factor for a (d,p) reaction involving a single value of l, s, and j can be written as

$$S=\frac{2J_i+1}{2J_f+1}\frac{(d\sigma/d\Omega)_{\text{exp}}}{\sigma(\theta)_{th}}.$$

The quantity $(d\sigma/d\Omega)_{\text{exp}}/\sigma(\theta)_{\text{th}}$ is the ratio of the experimentally determined differential cross section to that calculated from direct reaction theory. In the present investigation, $\sigma(\theta)_{\text{th}}$ was calculated using the Oak Ridge

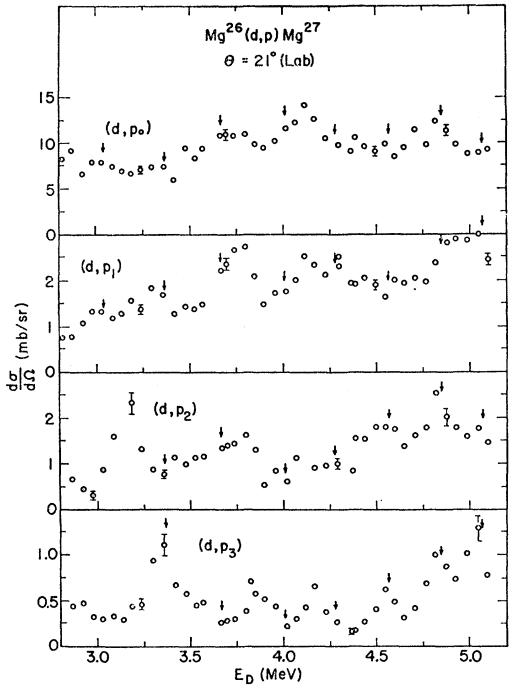

FIG. 8. Excitation functions for the $Mg^{26}(d,p)Mg^{27}$ reactions. The arrows indicate the energies at which angular distributions were measured.

code JULIE. The details and inherent approximations of this code have been discussed elsewhere.[7]

A comparison of $\sigma(\theta)_{\text{th}}$ with the experimental cross section is shown in Fig. 9 for the transitions proceeding to the ground and first two excited states. The transition to the third excited state is quite weak. The angular distributions appear to correspond to either $l_n=3$ or $l_n=4$, but the uncertainty in the fit and the strong energy dependence observed experimentally make the extraction of quantitative information extremely difficult.

C. Cutoff Radii

The assumption is made that the reaction in question takes place at the nuclear surface. If the exact wave functions were known, an evaluation of the matrix element of the interaction over the internal states would therefore show a very small contribution from the interior of the nucleus. This condition is approximated by neglecting any contribution to the matrix element for radii less than some arbitrary radius R_{LC}. This parameter is called the lower cutoff radius and turns out to be quite important in determining the ground state spectroscopic factor for the reaction under discussion. The $l=0$ bound-state wave function used in JULIE (truncated harmonic oscillator) has its primary maximum well inside the nuclear surface. The magnitude of the cross section predicted using this bound-state function is quite sensitive to the extent to which the interior of the nucleus is included in the calculations.

The proper value of R_{LC} cannot, in the present case, be chosen by comparison with the experimental data. Figure 10 shows a comparison between the data and the calculated cross sections for $R_{LC}=0$ (no cutoff) and $R_{LC}=4.8$ F (outside the nuclear surface). Note that both values of the cutoff radii give equivalent fits to the experimental data, although the spectroscopic factors for the ground-state transition differ by nearly a factor of 2. For an $l=2$ transition the dependence on R_{LC} would be expected to be weaker. This expectation is verified by the observation that the spectroscopic factor

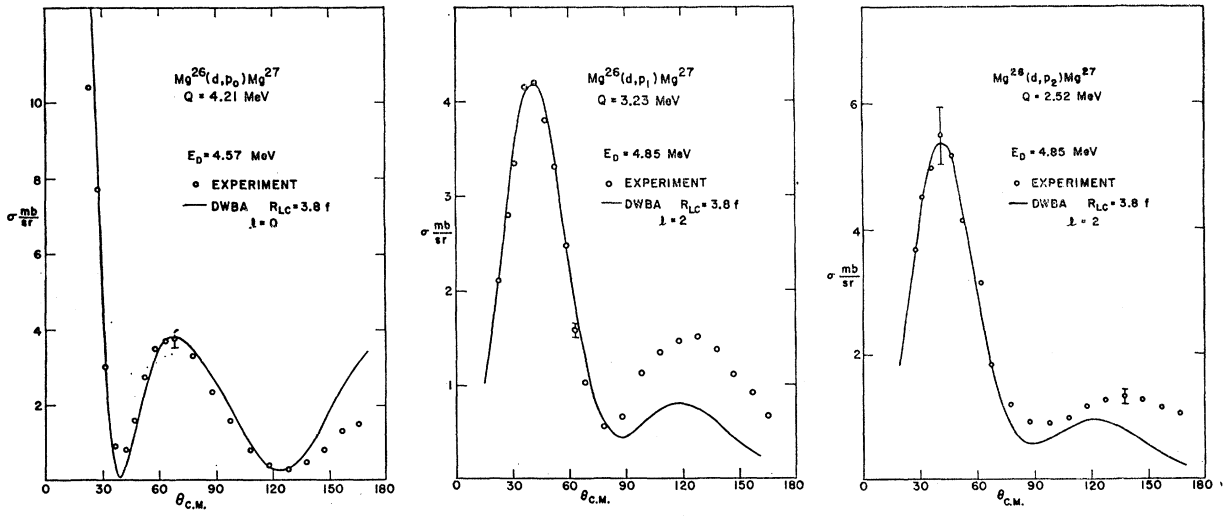

FIG. 9. Typical comparisons of the DWBA calculations with experiment.

[7] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report ORNL-3240, 1962 (unpublished); R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. 128, 2693 (1962).

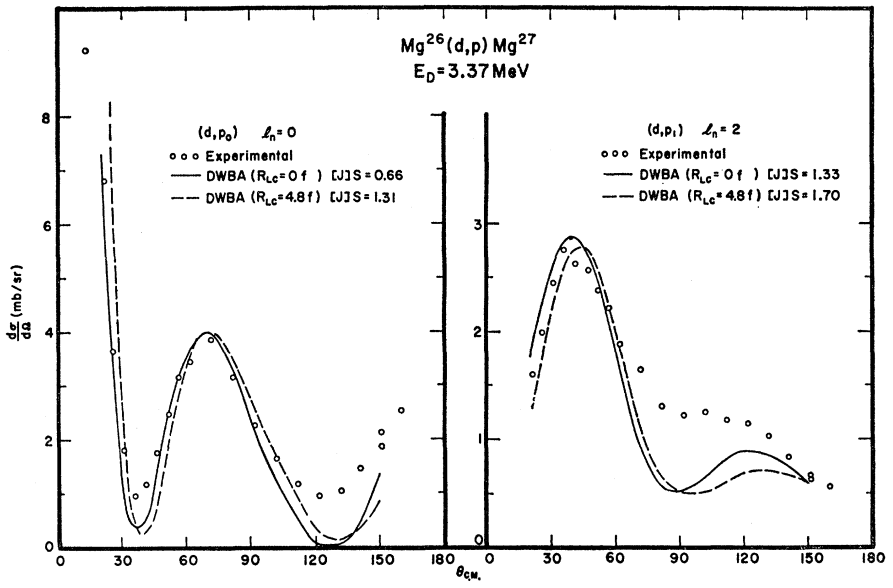

FIG. 10. DWBA calculations using different cutoff radii. In the $l=0$ case the spectroscopic factor differs by a factor of 2 for $R_{LC}=0$ and 4.8 F, although the shapes of the angular distributions calculated for these radii are very similar.

for the first excited state changes by 20% for $R_{LC}=0$ and 4.8 F (Fig. 10). The variation of S with R_{LC} is illustrated in Fig. 11 where the values of $(2J+1)S$ for the ground and first excited states as well as the ratio of the values for the two states are plotted as a function of the cutoff radius.

The value of $R_{LC}=3.8$ F was chosen for the results quoted in the next section. This choice was based on the assumption that a $2s_{1/2}$ or $1d_{3/2}$ neutron should be captured at the nuclear surface.

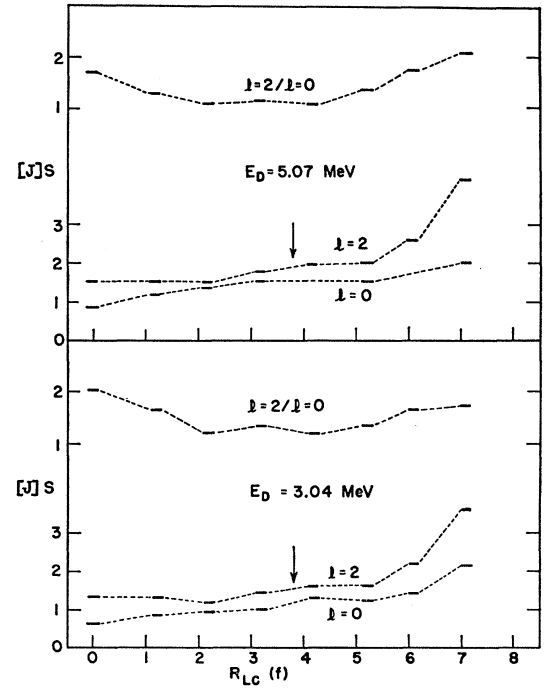

FIG. 11. Plot of $(2J+1)S$ as a function of R_{LC}. The $l=0$ curve is for the ground state of Mg^{27}; the $l=2$ curve for the first excited state of Mg^{27}. For clarity the ratio $(2J+1)S_{1st}/(2J+1)S_{gnd}$ is also plotted. The results are similar for two different deuteron bombarding energies.

V. SPECTROSCOPIC FACTORS

The results of the present DWBA calculations are summarized in the first half of Table II. Each of the spectroscopic factors listed is an average over the eight bombarding energies investigated. This averaging procedure was adopted in an attempt to smooth out any compound nuclear effects in the observed cross sections. The quoted uncertainties of 50% are due in part to the experimental errors, and in part to the ambiguities in the DWBA calculations, namely the effects of uncertainties in the various parameters employed in the calculations.

In the second half of Table II the ratios of the excited-state spectroscopic factors to that of the ground state are compared to the results of previous investigations. The ratios rather than the absolute values are chosen for the comparison since the earlier analyses were done with the plane-wave Born approximation and have large uncertainties in the absolute normalization. To make the comparison it was necessary to renormalize the previous results. The values quoted were in terms of θ^2 rather than S, where

$$S=\theta^2/\theta_0^2.$$

The single-particle reduced widths θ_0^2 which were used for the renormalization were obtained from Macfarlane's[2] analysis of the $Mg^{24}(d,p)Mg^{25}$ reaction, and are given by

$$\theta_0^2(1d)/\theta_0^2(2s)\approx 0.75.$$

The agreement with the present results is very good, considering the magnitude of the uncertainties involved.

TABLE II. Comparison of spectroscopic factors determined for Mg^{27}. Column (a) refers to the present work; column (b) to W. C. Parkinson and J. R. Maxwell, Phys. Rev. **126**, 1160 (1962); column (c) to J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 258 (1953); column (d) to S. Hinds, R. Middleton, and G. Parry, Proc. Phys. Soc. (London) **71**, 49 (1958). The renormalization of previous work is described in the text.

				$(2J+1)S/(2J+1)S_{\text{gnd}}$			
E_x (MeV)	l_n	J^π	$(2J+1)S$	$E_D=3.0-5.11$ (a)	$E_D=7.8$ (b)	$E_D=8.0$ (c)	$E_D=8.9$ (d)
0	0	$\frac{1}{2}^+$	1.4±0.7	1	1	1	1
0.99	2	$(\frac{3}{2},\frac{5}{2})^+$	1.6±0.8	1.15	1.2	1.3	1.4
1.69	2	$(\frac{3}{2},\frac{5}{2})^+$	1.3±0.7	0.9	0.7	···	···

If the assumption is made that the spin assignments of the first and second excited states are, respectively, $\frac{3}{2}^+$ and $\frac{5}{2}^+$, then the absolute spectroscopic factors can be compared with the predictions of the rotational model. Macfarlane[2] has calculated the absolute widths for the low-lying $K=\frac{1}{2}$ band in Mg^{25}. If the distortions of Mg^{27} and Mg^{25} are similar, then these values should qualitatively agree with those for the corresponding levels in Mg^{27}. This comparison can only be qualitative because of the differences in residual two-body interactions for the two nuclei.[2,8] In Table III the comparisons are made and are quite reasonable.

TABLE III. A comparison of the spectroscopic factors for the first three excited states with those for the $K=\frac{1}{2}$ band in Mg^{25} (both experimental and theoretical).

Mg^{27}			Mg^{25} $K=\frac{1}{2}$			
E_x (MeV)	J^π	S_{exp}	E_x (MeV)	J^π	S_{calc}	S_{exp}
0	$\frac{1}{2}^+$	0.70	0.58	$\frac{1}{2}^+$	0.46	0.50
0.987	$\frac{3}{2}^+$	0.40	0.98	$\frac{3}{2}^+$	0.15	0.25
1.69	$\frac{5}{2}^+$	0.22	1.96	$\frac{5}{2}^+$	0.07	0.15

VI. SUMMARY

The purpose of the present investigation was to extract spectroscopic information concerning the low-lying levels in Mg^{27}. Angular distributions were measured at several deuteron energies in an attempt to minimize the experimental uncertainties due to compound nuclear processes. With the exception of the third-excited-state transition, satisfactory fits to the angular distributions were obtained by calculations with DWBA. The sensitivity of these fits to the use of a cutoff radius in the calculations was not sufficient to determine this parameter unambiguously. This uncertainty contributed to a relatively large uncertainty in the extracted spectroscopic factors.

The ratios of the spectroscopic factors obtained agree with earlier results. A comparison with the experimental and theoretical values for the $K=\frac{1}{2}$ band in Mg^{25} shows a qualitative similarity, suggesting that the distortion for these two nuclei may be comparable.

ACKNOWLEDGMENTS

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The authors also wish to thank R. Horoshko for his help in performing the experiment, and Miss J. Liftin for her work in various phases of the computer operation.

[8] D. M. Brink and A. K. Kerman, Nucl. Phys. **12**, 314 (1959).